Engineering Notes

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Boundary-Layer Formation for Constant Accelerated Motion of an Axially Symmetrical Body

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Introduction

THE process of boundary-layer formation in two-dimensional flow for the case of constant acceleration of the body has been discussed by Schlichting.[1] As far as the authors know, there is no published analysis which discusses the boundary-layer formation in an axially symmetrical body in constant acceleration by the method of successive approximations up to the third order. In this Note, the boundary-layer formation of an axially symmetrical body in constant accelerated motion is studied by the method of successive approximations. The second and third approximation equations are derived and calculated by numerical method. The beginning of separation of a sphere in constant accelerated motion is compared with different orders of approximation.

The Equations of Motion and Solutions

The fundamental equations for non-steady laminar boundary layers over an axially symmetrical body are of the forms

$$\frac{\partial ru}{\partial x}+\frac{\partial rv}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

and the boundary conditions are $y=0$: $u=v=0$; $y=\infty$, $u=U(x,t)$, where x and y are distances measured along and normal to the surface of the body, r is the radius of the body of the section under consideration, u and v are the velocity components in x and y directions, and U is the potential velocity. Assuming that the velocity is composed of three terms

$$u(x,y,t)=u_0(x,y,t)+u_1(x,y,t)+u_2(x,y,t)$$

$$v(x,y,t)=v_0(x,y,t)+v_1(x,y,t)+v_2(x,y,t)$$

We can obtain the first, second, and third approximation equations which are

$$\frac{\partial u_0}{\partial t}=\frac{\partial U}{\partial t}+\nu\frac{\partial^2 u_0}{\partial y^2} \tag{3}$$

$$\frac{\partial u_1}{\partial t}+u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}=U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u_1}{\partial y^2} \tag{4}$$

$$\frac{\partial u_2}{\partial t}+u_0\frac{\partial u_1}{\partial x}+u_1\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_1}{\partial y}+ v_1\frac{\partial u_0}{\partial y}=\nu\frac{\partial^2 u_2}{\partial y^2} \tag{5}$$

and the boundary conditions are $y=0$: $u_0=u_1=u_2=v_0=v_1=v_2=0$; $y=\infty$: $u=U(x,y)$; $u_1=u_2=0$. The potential velocity of the body in constant accelerated motiom can be written as $t\leq 0$: $U(x,t)=0$; $t>0$: $U(x,t)=tw(x)$. Assuming the stream function including very possible derivative of w and r with respect to x of the form

$$\begin{aligned}\varphi(x,y,t)=2(\nu t)^{1/2}\{&rwt\zeta_0(\eta)+t[rwt\frac{\mathrm{d}(wt)}{\mathrm{d}x}\zeta_{1a}(\eta)+\\ &w^2t^2\frac{\mathrm{d}r}{\mathrm{d}x}\zeta_{1b}(\eta)]+t^2[r(wt)^2\frac{\mathrm{d}^2wt}{\mathrm{d}x^2}\zeta_{2a}(\eta)+\\ &(wt)^2\frac{\mathrm{d}r}{\mathrm{d}x}\frac{\mathrm{d}(wt)}{\mathrm{d}x}\zeta_{2b}(\eta)+(wt)^2\frac{\mathrm{d}^2r}{\mathrm{d}x^2}\zeta_{2c}(\eta)+\\ &rwt(\frac{\mathrm{d}wt}{\mathrm{d}x})^2\zeta_{2d}(\eta)+\frac{1}{\mathrm{r}}(wt)^3(\frac{\mathrm{d}r}{\mathrm{d}x})^2\zeta_{2e}(\eta)]\}\end{aligned} \tag{6}$$

where $\eta=[y/2(\nu t)^{1/2}]$. Then the velocity components become

$$\begin{aligned}u=&wt\zeta_0'+t[wt\frac{\mathrm{d}wt}{\mathrm{d}x}\zeta_{1a}'+w^2t^2\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x}\zeta_{1b}+\\ &t^2[w^2t^2\frac{\mathrm{d}^2wt}{\mathrm{d}x^2}\zeta_{2a}+w^2t^2\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x}\frac{\mathrm{d}(wt)}{\mathrm{d}x}\zeta_{2b}+\\ &w^3t^3\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x}\zeta_{2c}'+wt\frac{\mathrm{d}wt}{\mathrm{d}x}\zeta_{2d}'+w^3t^3(\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x})^2\zeta_{2e}'\end{aligned} \tag{7}$$

$$\begin{aligned}v=-2(\nu t)^{1/2}\{&t\frac{\mathrm{d}w}{\mathrm{d}x}\zeta_0+\frac{wt}{r}\frac{\mathrm{d}r}{\mathrm{d}x}\zeta_0+[\frac{wt^3}{r}\frac{\mathrm{d}r}{\mathrm{d}x}\frac{\mathrm{d}w}{\mathrm{d}x}+\\ &t^3(\frac{dw}{\mathrm{d}x})^2+t^3w\frac{\mathrm{d}^2w}{\mathrm{d}x^2}]\zeta_{1a}+\\ &[2wt^3\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}x}\frac{\mathrm{d}w}{\mathrm{d}x}+w^2t^3\frac{1}{r}\frac{\mathrm{d}^2r}{\mathrm{d}x^2}]\zeta_{1b}\}\end{aligned} \tag{8}$$

Inserting Eq. (7) into Eq. (3) we obtain a differential equation of the first approximation

$$\zeta_0'''+2\eta\zeta_0''-4\zeta_0'=-4 \tag{9}$$

and the boundary conditions are $\eta=0$: $\zeta_0=\zeta_0'=0$; $\eta=\infty$: $\zeta_0'=1$

Equation (9) has the same form as the constant acceleration case for two dimensional flow which has been solved by Blasius.[1] Inserting Eq. (7) and Eq. (8) into Eq. (4) we obtain

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two differential equations of the second approximation. They are of the forms

$$\zeta'''_{1a}+2\eta\zeta''_{1a}-12\zeta'_{1a}=4(-1+\zeta_0'^2-\zeta_0\zeta''_0) \tag{10}$$

$$\zeta'''_{1b}+2\eta\zeta''_{1b}-12\zeta'_{1b}=4\zeta''_0\zeta_0 \tag{11}$$

with the boundary conditions $\eta=0,\ \zeta_{1a}=\zeta'_{1a}=\zeta_{1b}=0;\eta=\infty,\ \zeta'_{1a}=\zeta'_{1b}=0.$ ζ_{1a} is identical with the second approximation equation of two dimensional case which has also been solved by Blasius.[1] ζ_{1b} has been calculated by computer. Its shape is shown in Fig. 1. The initial slope of ζ'_{1b} is $\zeta''_{1b}(0)=0.0904608$.

Inserting Eq. (7) and Eq. (8) into Eq. (5) we obtain five differential equations of the third approximation

$$\zeta'''_{2a}+2\eta\zeta''_{2a}-20\zeta'_{2a}=-4(\zeta_{1a}\zeta''_0-\zeta'_{1a}\zeta'_0)$$

$$\zeta'''_{2b}+2\eta\zeta''_{2b}-20\zeta'_{2b}=-4(\zeta_{1a}\zeta''_0+2\zeta_{1b}\zeta''_0-3\zeta'_{1b}\zeta'_0 + \zeta''_{1b}\zeta_0+\zeta''_{1a}\zeta_0)$$

$$\zeta'''_{2c}+2\eta\zeta''_{2c}-20\zeta'_{2c}=-4(\zeta_{1b}\zeta''_0-\zeta'_{1b}\zeta'_0)$$

$$\zeta'''_{2c}+2\eta\zeta''_{2d}-20\zeta'_{2d}=-4(\zeta_{1a}\zeta''_0-2\zeta'_{1a}\zeta'_0+\zeta''_{1a}\zeta_0)$$

$$\zeta'''_{2e}+2\eta\zeta''_{2e}-20\zeta_{2e}=-4(\zeta'_{1b}\zeta'_0+\zeta''_{1a}\zeta_0) \tag{12}$$

with the boundary conditions $\eta=0,\ \zeta_{2a}=\zeta_{2b}=\zeta_{2c}=\zeta_{2d}=\zeta_{2e}=\zeta'_{2a}=\zeta'_{2b}=\zeta'_{2c}=\zeta'_{2d}=\zeta'_{2e}=0;\ \eta=\infty,\ \zeta'_{2a}=\zeta'_{2b}=\zeta'_{2c}=\zeta'_{2d}=\zeta'_{2e}=0.$

Equation (12) has been calculated by computer. The function ζ'_{2a}, ζ'_{2b}, ζ'_{2c}, ζ'_{2d}, and ζ'_{2e} are shown in Fig. 2. The initial slopes of the five functions are

$$\zeta''_{2a}(0)=-0.0218086$$
$$\zeta''_{2b}(0)=-0.00333173$$
$$\zeta''_{2c}(0)=-0.00589340$$
$$\zeta''_{2d}(0)=-0.0636076$$
$$\zeta''_{2e}(0)=-0.0101829$$

The velocity gradient can be evaluated from Eq. (7)

$$\frac{\partial u}{\partial y}=\frac{wt}{2(\nu t)^{1/2}}\Big[\zeta''_0+t^2\frac{dw}{dx}\zeta''_{1a}+\frac{wt^2}{r}\frac{dr}{dx}\zeta''_{1b}+ wt^4\frac{d^2w}{dx^2}\zeta''_{2a}+\frac{wt^4}{r}\frac{dr}{dx}\frac{dw}{dx}\zeta''_{2b}+w^2t^4\frac{1}{r}\frac{dw}{dx^2}\zeta''_{2c}+ t^4\Big(\frac{dw}{dx}\Big)^2\zeta''_{2d}+w^2t^4\frac{1}{r^2}\Big(\frac{dr}{dx}\Big)^2\zeta''_{2e}\Big] \tag{13}$$

For a sphere with radius R the potential velocity is

$$U(x,t)=tw(x)=t(3/2)a\sin(x/R)$$

and $r=R\sin(x/R)$. It is known that separation occurs at a downstream stagnation point. Substituting the above relations and the initial slopes of the various functions in Eq. (13), we obtain separation time t_s from the following relation

$$2.256758-1.05212(3/2)(a/R)t_s^2 - 0.05675643[(3/2)(a/R)t_s^2]^2=0$$

Thus $t_s=1.1374(R/w)^{1/2}$. If only the second approximation is considered the separation time is $t_s=1.1950(R/a)^{1/2}$ which is about 5% larger than the value given by the third approximation.

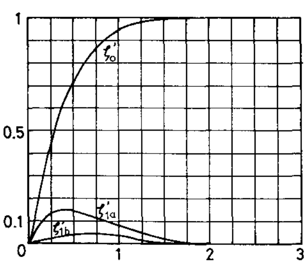

Fig. 1 The functions ζ'''_0, ζ'''_{1a} and ζ'''_{1b} for the velocity distribution in the nonsteady boundary layer

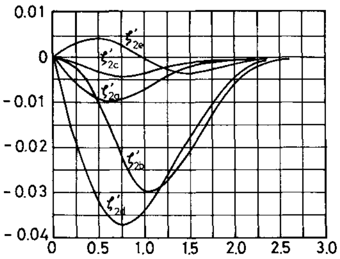

Fig. 2 The functions ζ'''_{2a}, ζ'''_{2b}, ζ'''_{2c}, ζ'''_{2d} and ζ'''_{2e} for the velocity distribution in the nonsteady boundary layer.

References

[1] Schlichting, H., *Boundary Layer Theory,* 6th ed., McGraw-Hill, New York, 1968, p. 406.

Leading-Edge Rotating Cylinder for Boundary-Layer Control on Lifting Surfaces

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Nomenclature

C_p = surface pressure coefficient
G = gap between fixed and moving surfaces (in.)
R = cylinder radius (in.)
Re = Reynolds number based on U_∞ and R
U_∞ = freestream velocity
U_c = cylinder surface velocity
X = dimensionless distance from leading edge of body

Introduction

THE use of boundary-layer control to increase the maximum lift on an airfoil has been well documented by many investigators and methods such as suction, blowing, vortex generators, turbulence promotion, etc., have found practical application in many situations. The use of a moving wall for boundary-layer control has been under investigation for many years, for example Refs. 1–3. However, until recently no serious attempts have been made to utilize the moving wall concept.

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